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This Journal is dedicated to the following aims:

1. Through published standard papers on the culture aspects, humanism and history of mathematics to deepen and to widen public interest in its values.

2. To supply an additional medium for the publication of expository mathematical articles.

3. To promote more scientific methods of teaching mathematics.

4. To publish and to distribute to groups most interested high-class papers of research quality representing all mathematical fields.

The National Council of Teachers of Mathematics

The seventeenth annual meeting of the National Council of Teachers of Mathematics was held in St. Louis, Missouri, December 31, 1935 to January 1, 1936, being the first annual meeting held with the A. A. A. S. One hundred and eighty-four registered for the meetings. A joint session with Section A of the A. A. A. S., the American Mathematical Society, and the Mathematical Association of America, was held on Tuesday morning, December 31. Professor Kenneth P. Williams of Indiana University presented a temporary report of the Joint Commission on the Place of Mathematics in the Secondary School. "The Main Purposes and Objectives in Teaching High School Mathematics" was discussed by William Betz of Rochester, New York, for the National Council, and W. W. Hart, for the Mathematical Association of America. On Tuesday afternoon, the Council presented a Symposium on the Teaching of Geometry. Professor W. H. Roever of Washington University, St. Louis, discussed "The Purpose, Nature, and Use of Pictures in the Teaching of Solid Geometry". John T. Rule, Clayton, Missouri, presented a paper on "Stereoscopy as an Aid to the Teaching of Solid Geometry". Rolland R. Smith, Springfield, Massachusetts spoke on "Developing the Meaning of Demonstration in Geometry". On Tuesday evening, an address of welcome was made by the Rev. Father Robert S. Johnston, President of St. Louis University, Miss Edith Woolsey of Minneapolis, Minnesota, responding. Professor Edwin W. Schreiber, State Teachers College, Macomb, Illinois, presented an illustrated lecture on "The History of the Development of the Metric System". Miss Ruth Lane, University High School, Iowa City, Iowa, presented a paper on "Mathematical Recreations, an Aid or a Relief?" Professor H. E. Slaught of the University of Chicago was elected Honorary President of the National Council. New Officers: President, Miss Martha Hildebrandt, Proviso Township High School, Maywood, Illinois; Second Vice-President, Miss Mary Kelly, Wichita, Kansas; three new members of the Board of Directors, E. R. Breslich, Chicago, Illinois, Leonard D. Haertter, Clayton, Missouri, and Virgil S. Mallory, Montclair, New Jersey. Two papers: "Functional Thinking and Teaching in Secondary School Mathematics" by Professor H. C. Christofferson, Miami University, Oxford, Ohio, and "The Crisis in Mathematics—at Home and Abroad" by Professor William D. Reeve. Professor Raymond Clare Archibald gave an address on "Babylonian Mathematics".

EDWIN W. SCHREIBER, *Secretary.*

The Scholar's Arithmetic

By E. R. SLEIGHT
Albion College

Immediately following the Revolutionary War there appeared in America a deluge of Arithmetics, written by American authors, as "Indispensible Aids to the Learners." One of this number, The Scholar's Arithmetic, written by Daniel Adams, towered above the mass of contemporary texts, and was in use far into the nineteenth century.

Daniel Adams was a native of Massachusetts and was graduated from Dartmouth College in 1797. He was a man of many interests, but found his greatest enjoyment in the study of arithmetic. The first edition of his work appeared in 1801 under the title of The Scholar's Arithmetic, or the Federal Accountant—"the Whole in a form and Method altogether New, for the Ease of the Master and the Greater Progress of the Scholar." The book was held in high esteem, as is shown by the type of recommendations which were given to it. A letter from the Reverend Laban Ainsworth to the publishers of the fourth edition, dated August 3, 1807, contains the following significant testimonial: "The superiority of the Scholar's Arithmetic to any other book of the kind in my knowledge clearly appears from its good effect on the schools I annually visit. Previous to its introduction, arithmetic was learned and performed mechanically; since, scholars are able to give a rational account of the several operations in arithmetic, which is the best proof of their having learned to a good purpose."

The aims of the book are expressed by the author in his preface to the third edition. This preface is "Dedicatory to School Masters" and is unusually interesting, not only because it points out the purposes of the book, but also because it shows clearly the scholarly mind of the author when he writes, "Through the whole book it has been my greatest care to make myself intelligible to the Scholar; such rules and remarks as have been compiled from other authors are included in quotations; the examples, many of them, are extracted from other authors. This I have not hestitated to do when I found them suited to my purpose.

"To answer the several intentions of this work, it will be necessary that it shall be put into the hands of every arithmetician: the blank

after each example is designed for operation by the Scholar, which being first wrought on slate, or waste paper, he may afterwards transcribe into the book.

"The supplement to each rule in this book is a novelty. I have often seen books with questions and answers, but in my humble opinion, it is no evidence that the Scholar comprehends the principles of that science which is his study, because that he may be able to repeat verbatim from his book the answer to a question on which his attention has been exercised two or three hours to commit to memory. Study is of little advantage to the human mind without reflection. To force the Scholar into reflection of his own, is the object of these questions unanswered, at the beginning of each supplement.

"Demonstrations of the reasons and nature of the operations in the extraction of the Square and Cube Roots have never been attempted, in any work before, to my knowledge. I hope that these will be found to be satisfactory."

The preface is followed by some explicit directions to Scholars. "Deeply impress your mind with the sense of the importance of arithmetical knowledge. The great concerns of life can in no way be conducted without it. Do not, therefore, think any pains too great to be bestowed for so noble an end. Drive far from you idleness and sloth; they are great enemies to improvement. Remember that youth, like the morning, will soon be past and the opportunities, once neglected, can never be regained. As much as possible endeavour to do everything by yourself; one thing found out by your own thought and reflection will be of more real use to you than twenty things told by an instructor."

An analysis of the text discloses the fact that it is divided into five parts. Part one is devoted to the "Fundamental Rules of Arithmetic," while part two contains "Rules essentially necessary for every person to fit and qualify himself for the transactions of business." Part three is entitled "Rules occasionally useful to men in particular employments of life," and in part four are to be found a "Miscellaneous Collection of questions, the purpose of which is to lead the Scholar into reflections concerning the foregoing principles." Part five is merely a collection of forms for writing notes, receipts, orders, deeds, indentures, and wills.

At the beginning of the study of the fundamental rules, we find rather an elaborate discussion of notation and enumeration. Here is shown the advantage of the Arabic numerals over the more cumbersome Roman notation. Also we are told that "For the greater ease in

reckoning it is convenient, in public offices and by men of business, to divide any number into periods consisting of six digits separated by the dot, and these into half periods separated by the comma. Thus, 521,768.532,467." Having discovered this use for the dot, one is lead to wonder in what manner the author indicated decimal numbers. We find that the comma, called a separatrix, placed high up on the number is used as in the case of 35'26, instead of in our modern 35.26. In his New Arithmetic, which appeared in 1831, the comma became inverted as a separatrix, and this same number was written 35‘26. Other interesting symbols are found in the text such as inverted parenthesis to denote multiplication, as in 3)(5 instead of 3×5. The dollar sign, with one vertical stroke instead of two, is used a few times.

To impress one with the magnitude of numbers this illustration is found. "If a person employed in telling money reckons an hundred pieces a minute, and he continue ten hours each day, it will take him nearly 17 days to reckon a million. And if we suppose the whole earth to be peopled as Britain, and to have been so from the creation, and that the whole race of mankind had constantly spent their time telling from a heap consisting of a quadrillion of pieces, they would hardly yet have reckoned a thousandth part of the quantity."

To obtain practice in computation, many examples are given, and each is "worked out by the Scholar on slate or waste paper, then transcribed to the book." In fact we have an excellent example of a modern work book. The particular copy of the sixth edition which I have examined, seems to have been the property of John Putnam,* as this name appeared on the title page, and comparing the abbreviation for "answer" which he used at the close of each example, leads one to believe that Mr. Putnam was the original owner.

The author suggests certain types of proofs, which are followed out very carefully in the examples solved in the book. For addition Adams suggests that "the upper line of figures be cut off. Then adding the lower lines which remain, place the amount under the amount first obtained by addition of all the sums, observing carefully that each figure falls directly under the column which produced it; then add this last amount to the upper line which you cut off. If the two sums thus obtained agree, then the problem is correct."

*Written, John Putnam's Book.

The following is an illustration of the rule, and will serve to interpret it:

$$\begin{array}{r} 37652 \\ \hline 21304 \\ 80163 \\ 25321 \\ \hline 164440 \\ \hline 126788 \\ \hline 164440 \text{ Proof.} \end{array}$$

This illustration is taken directly from the text.* The line below the first number indicates that "the upper line of figures has been cut off." The remaining three numbers have been added giving 126,788. This sum added to the number cut off gives 16440, as was found in the original operation of adding the four numbers. Every example in addition is proved by this method, omitting the last operation. Evidently the owner of the book performed this addition mentally, a process just coming into use at the time of the publication of the Scholar's Arithmetic.

Rules for multiplication are clearly stated: then follows the suggestion that "before any progress can be made in this rule, the following table must be committed perfectly to memory." The table referred to is the multiplication table, the only table of its kind found in the book, a decided departure from many of the early arithmetics. The rule for casting out the nines is here given as a proof for multiplication, and a thorough discussion of the principles underlying the rule are given in a clear and concise manner.

In the author's dedicatory statement he stresses the value of the "Supplements to each Rule," as a means of "forcing the Scholar into reflections of his own." An examination of some of the questions found in these supplements will indicate the extent to which this aim was carried out. At the close of the section on addition we find such questions as:

1. What is simple addition?
2. Why do you carry for ten rather than some other number?
3. Of what use is addition?

*The erroneous sums 136,788 and 17440 are found instead of the correct ones, 126,788 and 16440.

The questions following fractions are more thought provoking. Among them we find:

1. What is signified by the denominator of a fraction?
2. How do decimal fractions differ from Vulgar Fractions?
3. How do cyphers placed at the left hand of a decimal fraction affect its value?

In each type of problem throughout the book we find the rules clearly stated, in common with other texts of that period. However, the questions at the close of each section seem to carry out the author's aim, and the idea was a decided step in advance, showing that Adams was concerned not only with results, but also with the principles underlying the operations.

Among the "Rules essentially necessary for every person to fit and qualify him for the transaction of business" are found vulgar fractions, decimal fractions, federal money, tables of exchange, simple and easy methods of casting interest on notes and bonds when partial payments are made, compound multiplication and division, and simple and double rule of three. In approaching the subject of decimal fractions, the author suggests that "a unit be divided into ten equal parts, and each of these parts into ten equal parts, etc. In this way the denominator of a decimal fraction will always be ten, one hundred, etc. For this reason we need not write the denominator, but instead it is only required to retain the numerator with a point, called the separatrix, at the left hand, to distinguish it from a whole number." As already indicated, the point used is the comma. The rules for the fundamental operations with decimal fractions are very clearly stated and are quite the same as are used today. Very little space is given to Vulgar fractions.

In the early editions of Pike's arithmetic, the first edition of which appeared in 1788, much space was given to the discussion of the money question. In Pike's book we find "Tables for Reducing the federal coin and the currencies of the several states. Also English, Irish, Canada, Nova Scotia, Livres Tournois, and Spanish dollars, each to the part of all others." Confusion still existed at the beginning of the nineteenth century, as is shown by the fact that "a table for reducing the currencies of the several United States to federal money" is found in the Scholar's Arithmetic. But there is no mention of the minted dollars from other countries, so it would seem that some of the currency difficulties were disappearing.

The Single Rule of Three is here referred to as the Rule of Proportion, and occupies a prominent place in the mind of the author as a means of solving problems.

Under section three are found "Rules occasionally useful," and among these rules is to be found an elaborate discussion concerning square and cube roots. The author claims great credit for his explanations, as is indicated in the preface. He introduces the subject by the statement that "To every number there is a root, although there are numbers whose precise root cannot be obtained. But by help of decimals, we can approximate toward those roots, to any necessary degree of exactness. Such roots are called surd roots, in distinction from those, perfectly accurate, which are called rational roots." For a radical sign he uses $\surd$ when there is one term only in the radicand. In case there is more than one term he makes use of the vinculum, but does not join it to the sign used in case of a monomial. Thus: $\surd\overline{3+5}$. His explanation for the method of extracting the square root of a number is geometrical, based on selecting a square whose area approximates the number in question. By adding rectangles to two sides of this square the trial divisor is found. Then the correction to this trial divisor turns out to be a square, which when properly placed in the figure, completes a new square whose area represents the number whose square root is to be extracted—the side of this square representing the root desired. By use of a cube, Adams builds up a very complete method for extracting cube roots. This explanation must have continued to be printed in arithmetics for many years, for in my early days of teaching it was still found in text books, and blocks made for the purpose of understanding more fully the various steps were much in demand.

The table of contents contains references to problems concerning the lever, axle, and screw. But in view of the fact that the whole subject of "Mechanical Powers" is disposed of in ten very simple examples, this part of the book is of very little interest.

Some of the early American Arithmetics were distinguished from the English texts which were in common use in the colonies, by the fact that "Pleasant and Diverting Questions" were ignored. Not so with the Scholar's Arithmetic. These are inserted for the purpose of making arithmetic more attractive. The antiquity of some of these questions causes them to be interesting. The first one that appears in this text is stated as follows:

As I was going to St. Ives,
I met seven wives,
Every wife had seven sacks,
Every sack had seven cats,
Every cat had seven kits,
Kits, cats, sacks, and wives,
How many were going to St. Ives.

This is but another form of Fabonaci's "Seven old women going to Rome" and with a similar problem found in Ahmes Papyrus, is undoubtedly the oldest known in all mathematical recreations.

In reviewing this arithmetic we have been impressed with the fact that the author was justified in his belief "that this work might prove a kind assistant towards a more speedy and thorough improvement of Scholars in Numbers, and at the same time relieve masters of a burden of writing out Rules and Questions, under which they have so long laboured, to the manifest neglect of other parts of their schools."

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A Discussion of the Methods of Science History, Art and Mathematics

By WALTER M. MILLER
Massachusetts State College

These four subjects can be divided roughly into two classes,—those which are essentially a record of facts, as science and history, and those which are imaginative and are a record of ideas, as art and mathematics. But such a classification is not a fine one, for the characteristics of the members of one class are found to some extent in the members of the other. This is especially noticeable when one considers the methods of each of these subjects.

The object of science is to discover or discern the so-called laws of nature, i. e. those uniformities which persist—the invariants, so to speak, of nature. Science is said to be exact, positive, and objective as opposed to the inexact, vague, and subjective. It conveys its ideas in defined, direct, and general terms as opposed to indirect, undefined, or symbolic terms. It rests on clear and precise knowledge as contrasted with opinion, belief, or faith. It deals with things or objects of thought that are common to a great many persons, accessible to everyone, so that its observations may be checked, re-examined, and verified, as opposed to those things which are personal or subjective, or which center in the individual mind and for which a proof is almost impossible. Mathematics, as one can see from this description, is, therefore, a science.

The procedure or method of a science is

1) the accumulation of data or facts,
2) the classification of the facts according to the end in view,
3) generalization,
4) the formation of hypotheses to explain the facts,
5) deduction from these hypotheses,
6) verification or comparison of conclusions with the observed facts.

The first step of this process is simply observation of phenomena. Such questions as whether there really exists an objective world which produces the phenomena, or whether the law of causation holds, or whether nature is uniform, are not properly subjects for the scientist to consider as scientists but rather they are the proper inquiries of the philosopher.

The classification and generalization of the facts is more important than the accumulation of the facts themselves. Often the person who is keen enough to notice similarities and likenesses which have escaped the notice of others, may have suggested to him, thereby, something which will be fruitful of results of great importance. In mathematics, this keenness is especially to be desired.

The forming of hypotheses to explain the facts is to reason inductively and may be considered as the most important step in the whole process. Facts however numerous or unusual are sterile and bare unless they may be explained. In forming hypotheses to account for phenomena the scientist must use all his powers of intuition and imagination. In so doing, he comes nearest to being an artist. The mathematician goes even farther. He creates, when necessary, new entities to explain troublesome facts or situations. The highest powers of the mind come into play at this point. It is a characteristic of the genius to be able to formulate many hypotheses quickly and often from very few facts. This quality of mind is especially noticeable and is indeed necessary in mathematicians.

The succeeding steps, deduction and verification, are checks on the hypotheses or theories offered. They are necessary to show that the actual facts are not at variance with any of the logical consequences derived from the proposed explanations.

This, briefly, is an outline of the scientific method. The method of history so much resembles this that it too may be classed as a science. The object of science is the discovery of the processes thru which things have come to be as they are. It is the problem of history to find out how man has come to be as he is, and to explain how the present has evolved from the primitive past, to reconstruct and describe the successive steps. History, then, may be described as that science which is concerned essentially not with things but with man. Whereas the physical sciences have their accurate formulae and natural laws giving accurate, measurable results derivable from them, history, since it deals with the variable human behavior, has difficulty in deducing certain results from the laws it formulates. The historian Buckle sought to extend to history the certainty which he attributed to other sciences. (Scientists are realizing more and more today how certain (!) their laws and methods are. It is one of the satisfactions of the mathematician to know that he need not feel uneasy on this score.) Moreover, history is an inverse science and its problems are frequently capable of several solutions. The more conditions are imposed on a problem, the fewer become the number of solutions; and if too many conditions be imposed, the solution becomes impossible. It is

the task of the historian to select or propose the most probable and the most acceptable solution to the problems that arise in his field.

In some respects history is like geology; its phenomena can not be repeated at will. But unlike the geologist, the historian seldom, if ever, has recourse to catastrophic explanations for some of the phenomena he observes. Rather is he guided in his methods by the principles of uniformity.

The methods of history are similar to those of the sciences,—

1) acquisition of the source material,—indirect observation,
2) criticism of the material,
3) establishment of the facts,
4) classification of the material—usually according either to time, geography, or subject matter,
5) generalization, i. e. construction of a drama of events from the facts.
6) interpretations, imagining and grouping the facts.

This requires a scientific not a poetic imagination. The poet may, if he wishes, create his materials but the historian must be bound by the facts of the case which confront him. Any gaps in the evidence must be filled by constructive scientific reasoning.

7) exposition to others.

Since all the facts cannot be described, those must be chosen without which the evolution of man cannot be described. In this last step the historian has his opportunity to be artistic. His method of exposition he may create for himself. Handicapped as he is in being unable to reproduce the phenomena with which he deals he is forced to rely on his intuition to see the solution of his problems.

Art differs from science in that it is concerned primarily with beauty and goodness while science is concerned with facts regardless of these qualities.

Someone has well expressed the difference by saying that "the poet picks the flowers not knowing their names and holds them out to us to our joy,—then the botanist comes and discovers what sort of plants they are."

Art tries to represent faithfully and unerringly character and beauty as they exist. It springs from the poetic yearnings and emotions suggested by aspiration after the true, the good, and the beautiful. It is the material means for conveying similar ideas appealing to the imagination,—music via sound; poetry via language; painting via color, light and shade; sculpture via form; etcetera.

How are these results achieved? Certainly not by deductive nor inductive methods, but by methods which must be called intuitive and creative. They are often the result of inspiration which, though momentary in itself, must be preceded by a long period of preparation and "incubation".

It is the mark of a strong mind to be able to recognize likenesses and to associate ideas. Such a mind is necessary for the scientist, but it is indispensable to the artist. The latter must be and is quick to see and to feel resemblances, to be sensitive to the most unusual likenesses. He must be able to seize ideas when they come and to create, if necessary, corresponding forms to express them. He must let his intuition guide him rather than his reason. Shakespeare for example, must have felt the effectiveness of the graveyard scene in Hamlet, rather than reasoned about it. The artist must obey those ideas or suggestions which come to him from his subconscious mind. Shelley believed that poetry acted in a "divine and unapprehended manner beyond and above consciousness." It is by intuition and creative inspiration that the great things in art have been achieved.

When one considers the methods of mathematics one finds that they combine both the methods of science and also of art. Indeed, the methods of science and of mathematics are so much alike in some respects that one can as well say science is mathematical in its processes as mathematics is scientific. The procedure in mathematics as in science is observation, comparison, classification, generalization, deduction.

The essential nature of mathematics shows itself in the discernment of likenesses and similarities in what are seemingly different domains. Great progress in mathematics depends largely on the discovery of methods of simplification, of unifying principles, of greater generalizations, and also incidentally of conciseness and elegance of treatment. Analogy is the great means for discovering and forming generalizations, though they may also be discovered by removing premises in arguments and noting consequences. Analogies between different theories often lead to developments in still other similar theories. Poincaré is the great example of a mathematician who generalized much.

It has been claimed by some people that in mathematics induction was completed long ago and that now mathematics is merely engaged in deductive and verifying processes. Fermat's last theorem, an example of inductive reasoning, has been proved true for a large number of cases though the general proof is still wanting. Many people think that this sort of work is the main function of mathematics.

But mathematics is much more than mere deductive reasoning. To be sure deductive methods have a large place in mathematics, but the chief use of deduction is to express results (found usually by other methods) simply and concisely so that they may be followed easily by others, and also to verify results obtained by any kind of mathematical thinking.

One of the chief ways of obtaining mathematical truths is by the intuitional method. In this respect the method of the mathematician is the method of the artist. Intuition is the taking in at a glance, without going through reasoning processes, the conditions and relations of a problem. By means of it the mathematician keeps in mind not only the problem itself but also the importance of every notion that appears in relation to it. It enables him to pass quickly from a few cases to general theorems and to grasp the essentials of a problem.

But the distintive mark of mathematics is its ability, when confronted by difficulties, to create new entities. This has been the ear mark of mathematics throughout history, as the notions of irrationals, negatives, imaginaries, ideals, non Euclidean geometry, hyperspaces, functional space, etc. bear witness. The creations of the mathematical mind are sometimes of a critical rather than of a constructive nature. But they are necessary in order to refine previous definitions or to show the limited range of applicability of some proposition. Examples of this sort of creation are the function of Weierstrass to show that a function may be continuous without having a derivative; and Darboux's function which is discontinuous throughout an interval though taking all values in the interval.

The mathematician, like the artist, must be sensitive to suggestions from whatever source they may come. He must have a keen mind to note resemblances and likenesses. He must not be averse to letting his intuition guide him, and must be ever alert to seize ideas which may come to him from his subconscious mind. In short, he must be truly an artist, for the creative method is the method par excellence. To acquire it, if indeed it can be acquired, he must "read widely, scrutinize intently, reflect profoundly, and watch alertly for the birth of new ideas."

Concerning the Base of the Natural System of Logarithms

By D. H. RICHERT
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Felix Klein proceeds from the principle that the proper source from which to bring in new functions is the quadrature of known curves. Details are left to the instructor.[1]

Following this principle he starts with the hyperbola $\xi\eta = 1$ and defines the logarithm of x as the area under the curve between the ordinates corresponding to $\xi = 1$ and $\xi = x$. This area is given by the formula

$$S = \int_1^x \frac{d\xi}{\xi} = \lg x. \tag{1}$$

Therefore equation (1), where x may take any positive values, defines the logarithm of x.

Applying the transformation $c\xi = \xi'$ to the variable of integration in (1), it is shown that the relation

$$\int_1^x \frac{d\xi}{\xi} = \int_c^{cx} \frac{d\xi}{\xi} \tag{2}$$

holds, from which follow the theorems

$$\lg a + \lg b = \lg(ab), \tag{3}$$

$$\lg(a/b) = \lg a - \lg b, \tag{4}$$

$$\lg(a)^n = n \lg a. \tag{5}$$

We wish to show how it follows from the definition (1) that e is the "base" of the logarithms thus obtained, where e is defined in the usual way, and the term "base" means the number whose logarithm is 1.

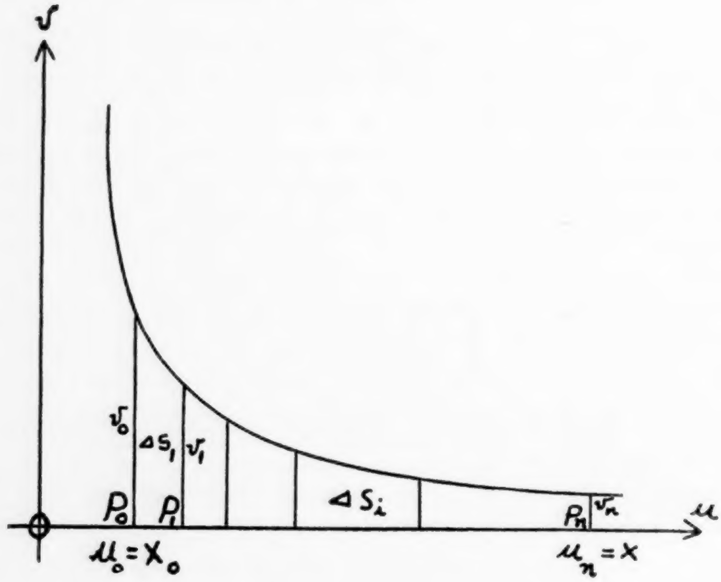
[1]Klein, "Elementarmathematik von Hoeheren Standpunkt aus", Leipzig, 1908, pp. 345-347.

See also the translation of this work by Hedrick and Noble, New York, 1932, p. 156.

Consider the area under the hyperbola $uv = 1$ between the u-axis and the ordinates v_0 and v_n corresponding to the abscissas u_0 and u_n respectively. (See figure).

Making use of a number m defined by

$$m = (x/x_0)^{1/n}, \tag{6}$$



divide the interval P_0P_n into n unequal parts. (See figure).

Set

$$\left\{\begin{array}{l} u_0 = x_0, \\ u_1 = mx_0, \\ u_2 = mu_1, \\ u_3 = mu_2, \\ \cdot \\ \vdots \\ u_n = mu_{n-1} = x. \end{array}\right. \tag{6'}$$

If $x > x_0$, it is true that $x_0 = u_0 < u_1 < \ldots < u_{n-1} < u_n = x$,

while if $x < x_0$, it is true that $x_0 = u_0 > u_1 > u_2 \ldots > u_{n-1} > u_n = x$.

The elements of area, ΔS_i, $(i=1, 2, 3, \ldots, n)$ trapezoidal in form, are all equal, for

$$\Delta S_1 = (u_1-u_0)\left(\frac{v_0+v_1}{2}\right) = \left(\frac{u_1-u_0}{2}\right)\left(\frac{1}{u_0}-\frac{1}{u_1}\right)$$

$$=\frac{u_1^2-u_0^2}{2u_0u_1}=\tfrac{1}{2}\left(\frac{u_1}{u_0}-\frac{u_0}{u_1}\right)$$

$$=\tfrac{1}{2}(m-1/m).$$

Similarly, on account of (6)′,

$$\Delta S_n=\tfrac{1}{2}\left(\frac{u_n}{u_{n-1}}-\frac{u_{n-1}}{u_n}\right)=\tfrac{1}{2}(m-1/m).$$

Denote the sum of these elements of area by S', then

$$S'=n/2(m-1/m).$$

Hence the area *S* under the curve is given by

$$\text{(7)}\qquad S=\lim_{n\to\infty}\left[\frac{n}{2}(m-1/m)\right]=\lim_{n\to\infty}\left[\frac{n}{2}\left\{\left(\frac{x}{x_0}\right)^{1/n}-\frac{1}{(\pm x_0)^{1/n}}\right\}\right]$$

When $x_0=1$ then

$$\text{(8)}\qquad S=\lim_{n\to\infty}\left[\frac{n}{2}(x^{1/n}-1/x^{1/n})\right].$$

Since *n* is large, $x^{1/n}$ differs but little from *1*, therefore we may write

$$\text{(9)}\qquad x^{1/n}=1+\Delta x,$$

so that

$$n=\frac{\lg x}{\lg(1+\Delta x)}$$

Making use of (9) and (10) in (8), and observing that as $n\to\infty$ $\Delta x\to 0$, we get after some reductions

$$\text{(11)}\qquad S=\lg x\cdot\lim_{\Delta x\to 0}\left[\frac{\Delta x}{\lg(1+\Delta x)}\right]$$

The limit of the second factor in the right member of (11) is *1*, as may be easily verified by applying to it de l'Hospital's theorem on indeterminate forms. Hence it follows from (11) that

$$\lim_{\Delta x \to 0} \left[\frac{\lg(1+\Delta x)}{\Delta x} \right] = 1, \tag{12}$$

or what amounts to the same thing,

$$\lg \left\{ \lim_{\Delta x \to 0} \left[(1+\Delta x)^{1/\Delta x} \right] \right\} = 1. \tag{13}$$

If in (13) we let $\Delta x = 1/n$, (13) becomes

$$\lg \left\{ \lim_{n \to \infty} \left[1 + \frac{1}{n} \right]^n \right\} = 1. \tag{14}$$

that is, $$\lg e = 1.$$

Hence *e* is the base of the system.

It should be observed that the limiting value of the last factor in (11) can be found by the theorem mentioned, but in our discussion this value follows directly from the definition of the logarithm of *x* given by (1), which says that

$$S = \lg x$$

Generalized Pythagorean Numbers

By DEWEY C. DUNCAN
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Occasionally one desires to find sets of "Pythagorean" or "right triangle" numbers; i. e., integers a, b, and c, such that $a^2+b^2=c^2$, examples being (3,4,5) and (5,12,13). In fact, one has formulas that yield *all* sets of such numbers, namely,

$$a=k(m^2-n^2), \qquad b=2\,k\,m\,n, \qquad c=k(m^2+n^2)$$

The object of this note is to exhibit the corresponding most general formulas that yield *n* numbers, $a_1, a_2, a_3, \ldots, a_n$, such that $a_1^2+a_2^2+a_3^2+\ldots+a_{n-1}^2=a_n^2$. It will be noted that the above formulas for "Pythagorean" numbers arise if one takes $n=3$, or if one takes $a_i=0$ for $i>3$. The derivation of the general formulas may be effected as follows:

Replace a_i/a_n by x_i; the equation, $a_1^2+a_2^2+\ldots+a_{n-1}^2=a_n^2$, becomes $x_1^2+x_2^2+x_3^2+\ldots+x_{n-1}^2=1$, the equation of the unit "hypersphere" with center at the origin. Now consider the ∞^{n-2}-fold system of lines that pass through the point $(-1,0,0,0,\ldots,0)$. Their equations are $x_i=t_i(x_1+1)$, $(i=2,3,4,\ldots,n-1)$. If t_i are all rational the corresponding line intersects the sphere again in a point whose coordinates are all rational; moreover, if a line of the system meets the sphere in such a rational point, the t_i are all rational. That is to say, if t_i take on all possible rational values one obtains all possible rational values of x_i satisfying the equation of the hypersphere. The coordinates of this other point of intersection of the line and sphere are readily found to be

$$x_1=\frac{1-(t_1^2+t_2^2+\ldots+t_{n-2}^2)}{1+(t_1^2+t_2^2+\ldots+t_{n-2}^2)},$$

$$x_j=\frac{2t_{j-1}}{1+(t_1^2+t_2^2\ldots+t_{n-2}^2} \quad (j=2,3,\ldots,n-1).$$

Since the various t_i are rational they may be written as p_i/q_i, where p_i and q_i are integers $(i=1,2,3,\ldots\, n-2)$. Making these replacements and expressing the results in terms of the original a_i, one has the

desired formulas. One observes that there are $2n-3$ arbitrary parameters, including the factor k. Although the values of all the a_i are integral, for the sake of symmetry certain fractional terms are retained. Accordingly,

$$a_1 = k\left[\prod_{i=1}^{n-2} q_i^2 \left\{ 1 - \sum_{i=1}^{n-2} (p_i/q_i)^2 \right\}\right]$$

$$a_j = k[\prod_{i=1}^{n-2} q_i^2(2p_j/q_j)], \quad (j=1,2,3,\ldots, n-3)$$

$$a_n = k\left[\prod_{i=1}^{n-2} q_i^2 \left\{ 1 + \sum_{i=1}^{n-2} (p_i/q_i)^2 \right\}\right]$$

(Throughout $i=1,2,3,\ldots, n-2$).

yield *all* possible values of a_i satisfying the relation

$$a_1^2+a_2^2+a_3^2+\ldots+a_{n-1}^2=a_n^2,$$

i. e., all *ratios* $a_1 : a_2 : a_3 : a_4 : \ldots : a_n$, from which all sets of Generalized Pythagorean Numbers may be obtained by removing a common factor. Thus the set (3, 2, 6, 7) cannot be directly obtained from the formulas for $n=4$, but is obtained by removing the common factor, 125, from the set (375,250,750,875) which the formulas yield. Although the ordinary Pythagorean Numbers are fairly abundant, seven sets of three numbers having no common factor occurring less than fifty, namely, (3,4,5), (5,12,13), (8,15,17), (7,24,25), (20,21,29), (12,35,37), and (9,40,41), the number of sets of Generalized Pythagorean Numbers increases very rapidly with increase of n; there are 82 such sets of four numbers less than fifty. They are the following:

(3,2,2,1)	(23,18,14,3)	(35,10,6,33)	(45,44,8,5)
(7,6,2,3)	(23,18,6,13)	(37,36,8,3)	(55,40,16,13)
(9,4,4,7)	(25,20,12,9)	(37,28,24,9)	(45,40,8,19)
(9,8,4,1)	(27,22,14,7)	(37,28,12,21)	(45,28,20,29)
(11,6,6,7)	(27,10,2,25)	(37,24,8,27)	(45,28,4,35)
(11,6,2,9)	(29,24,16,3)	(39,34,14,13)	(45,20,16,37)
(13,12,4,3)	(29,24,12,11)	(39,34,2,19)	(47,42,18,11)
(15,14,2,5)	(29,16,12,21)	(39,22,14,29)	(47,42,2,21)
(15,10,2,11)	(31,30,6,5)	(39,14,10,35)	(47,38,18,21)
(17,12,12,1)	(31,22,6,21)	(39,26,2,29)	(47,38,6,27)

(17,12,8,9)	(31,18,14,21)	(39,26,22,19)	(47,34,18,27)
(19,18,6,1)	(31,4,6,27)	(41,32,24,9)	(47,18,6,43)
(19,10,6,15)	(33,32,8,1)	(41,24,12,31)	(49,48,4,9)
(19,6,6,17)	(33,32,4,7)	(41,12,4,39)	(49,40,24,15)
(21,20,4,5)	(33,28,4,17)	(41,24,4,33)	(49,36,32,9)
(21,26,2,7)	(33,20,20,17)	(43,42,6,7)	(49,36,24,23)
(21,16,11,8)	(33,20,8,25)	(43,42,4,9)	(49,36,12,31)
(21,16,4,13)	(33,8,8,31)	(43,38,18,9)	(49,36,4,33)
(21,8,4,19)	(35,30,18,1)	(43,30,30,7)	(49,24,12,41)
(21,14,2,23)	(35,30,6,17)	(43,30,18,25)	
(23,22,6,5)	(35,26,18,15)	(43,18,4,39)	

Continuous Transformations of Finite Homogeneous Spaces*

By DOROTHY McCOY

Stephens† has studied the biunivocal continuous transformations of certain finite spaces where a function *f* determines a transformation of one space on another. We propose to study such spaces under the additional requirement that both spaces be homogeneous. This was suggested after noting that none of his transformations of the homogeneous space P*48*‡ resulted in homogeneous spaces. Some of the most interesting results of this paper come from the space P*48*. Properties of the cardinal number of the space play an unusual role here.

Let P and Q be two spaces such that the function *f* determines an application of the set P on the set Q. The function *f* is continuous in P for the element *a* of this space if for every subset G is included in P such that a is included in K(G)** one has the formula:

f(a) is included in $\{f(G-a)+K[f(G)]\}$. When the transformation is biunivocal this formula becomes f(a) is included in K[f(G)]. All the transformations studied in this article are biunivocal.

If for every two elements *a* and *b* of a space there exists a one to one bicontinuous transformation of the space into itself which transforms *a* into *b* the space is said to be topologically homogeneous.

The properties studied for these topologically homogeneous spaces are as follows:***

P1. Monotonic. For every A, B subsets of P such that A is included in B, K(A) is included in K(B).

*Presented to the American Mathematical Society December 28, 1934.

†Stephens, Rothwell; Continuous transformations of finite spaces. *The Tôhoku Mathematical Journal*, Vol. 39, Part I, p. 98-106, April, 1934.

‡The symbol P*48* stands for a combination in which all properties are present except 4 and 8. P38 stands for the combination of properties 3 and 8 with all other properties denied.

**A space in the sense used here is a system (P;K) composed of an aggregate P of elements p and a relation K among the subaggregates of the space. The space is finite if the number of elements of P is finite.

***Stephens, loc. cit. p. 98.

McCoy, Dorothy; The complete existential theory of eight fundamental properties of topological spaces. *The Tôhoku Mathematical Journal*, Vol. 33, Nos. 1, 2 (1930), pp. 89-116.

P2. Partition. For every partition of $E=A+B$ such that $A\neq O$, $B\neq O$, $K(E)$ is included in $K(A)+K(B)$.

P3. Accessible. For every E, $K(K(E))$ is included in $K(E)$.

P4. Non-singular. Every set of a single element has a null K-et.

P5. Uniqueness. For every p element of $K(E)$, there exists a unique family F composed of all A included in E such that p is an element of $K(A)$.

P6. Separable. There exists an enumerable set N such $P=N+K(N)$.

P7. Perfect. Every element is a K-point of at least one subset of the space.

P8. Connected. For every $A\neq O$, $B\neq O$ such that $P=A+B$, it is true that $A\overline{B}+B\overline{A}=O$ where $\overline{A}=A+K(A)$.

On a space (P;K) of a finite number of elements is defined another space (P;J) which is a continuous, one to one transform of (P;K). This is done by adding points of the space to the K-set in such a way as to preserve homogeneity. The elements of the space (P;K) are considered to be $p_1,p_2,p_3,\ldots,p_n$ where n is any finite integer larger than six as some special cases arise when fewer elements are used. These are indicated by Stephens and in my thesis. The transformation of any element *a* to any other element *b* to determine homogeneity is easily seen by thinking of the elements as arranged around a circle and the transformation as a clockwise rotation.

THE SPACE P*48*

The finite space P*48* has $K(E)=E$ for every E.* It is homogeneous, and homogeneity of a transformed space is assumed to be required throughout this paper. Note that all transformations of this space are singular, perfect and separable.

Theorem 1. *The uniqueness property requires $J(O)=O$ and the J-sets of single elements to be equal to their K-sets.*

Homogeneity requires $J(O)=O$ or P then the uniqueness property requires $J(O)=O$. Since $J(O)=O$ sets of single elements must have single elements J-sets for otherwise the points would not be distinguished by unique families of subsets.

Theorem 2. *The partition and uniqueness properties imply* $J(E)=K(E)$.

*Stephens, loc. cit. p. 103. Stephens proves that if a finite space has properties P*48* then $K(E)=E$: where E is any subset.

Theorem 1 shows this to hold for the null set and sets of single elements. Since we are using the basic space $K(E)=E$ we have $J(E)$ includes E. The partition property requires $J(E)$ included in $J(A)+J(B)$ where $E=A+B$. For sets of two elements say p_i+p_j we have both $J(p_i+p_j)$ includes p_i+p_j and $J(p_i+p_j)$ included in $J(p_i)+J(p_j)=p_i+p_j$ hence $J(p_i+p_j)=p_i+p_j$ for every set of two elements. This can then be extended to sets of three elements, of four elements and on to sets of n elements.

We have here that the partition and uniqueness properties imply $K(E)=K(A)+K(B)$. This is in addition to the usual situation where the monotonic and partition properties together imply this relation.*

Corollary 1. The partition and uniqueness properties imply that the space is accessible, monotonic and not connected.

Corollary 2. A connected space which has the uniqueness property has the negative of the partition property.

Theorem 2 shows that it is impossible to transform the space to a homogeneous one having the combinations of properties P*4*, P*34*, P*348*, P*14*, P*148*, P*134*, P*2567*. Stephens has shown that the two cases P*348* and P*14* do not occur where no limitation of homogeneity is required. My thesis shows that no homogeneous finite space has the combination P*14*.

Theorem 3. *The monotonic property, the negative of the uniqueness property, and the property of not connected implies the negative of the partition property.*

$J(O)=O$ or P. If $J(O)=P$ and the space is monotonic then $J(E)=P$ for every E and the space is connected, hence $J(O)=O$. The negative of the partition property implies $J(E)$ is not included in E either for sets E of single elements or for sets of more than one element. If $J(E)$ is not included in E for sets of single elements then the monotonic property implies the property of being connected since for the space to be homogeneous every set of a single element has added J-points when one set has. Hence sets of single elements have single element J-sets. If for sets of more than one element we have $J(E)$ is not included in E then the partition property is denied.

The combinations P*458* and P1267 are impossible on account of theorem 3. My thesis shows P1267 not possible for finite spaces.

With three properties invariant there could be not more than 32 different spaces distinguished by the other five properties. The preceding theorems show nine of the combinations impossible. By

*My thesis loc. cit. p. 91.

use of the following operators on the sets of space *P48*; homogeneous spaces are defined having added J-points and various combinations of the properties.

01. $J(E) = P$

02. If E contains p_i then J(E) contains p_{i+1} where the element after p_n is p_1.

03. If E includes $p_i p_{(i+1)}$ $p_{(i+2)}$ then J(E) includes (p_{i+4}) where the element after p_n is p_1.

S1. Null set.
S2. Sets of three elements.
S3. All subsets.
S4. Sets of $n-2$ or more elements.
S5. Sets of $n-1$ elements.
S6. Sets of $n/2-1$ or fewer elements.
S7. All sets except S5.
S8. Sets of more than one element.
S9. Sets of $n/2+1$ or more elements.

When operator 01 is applied to sets of the group S3 the resulting space has all eight properties except 4 and 5. This is written 01S3 P*45*. Similarly we have 02S3 P*345*; 01S5 P*248*; 01S8 P*245*; 01S4 P1367; 03S3 P1567; 02, 3S3 P1678; 02S9 P167; 01S7 P*145*; 01S1 P2367; 01S1,02S3 P2678; 02S6 P267; 03S2 P3567; 01S1,8 P3678; 01S1,03S2 P367; 02,3S3,01S1 P678; 03S3,01S1 P67. In addition the identical transformation has properties P*48* and the space having J-points added according to the following rule has properties P567. Each set with number of elements $\geqq n/2$ whose elements are of a consecutive sequence the J-set contains the next element. It is understood that the element following p_n is p_1.

The homogeneous space having properties P*24* appears not to be a transformation of the space P*48* although proof of this has not been found.

When the number of elements is required to be prime, spaces P*234*, P*14* and P5678 have been obtained. The following method of adding J-points is used to define these three spaces with a prime number of elements.

Consider all sets E of k elements where $1<k<n$ and choose one E containing p_1, p_1 being defined as the related element to this E. The set E contains the elements $p_1, p_{n_1}, p_{n_2}, \ldots, p_{n_{k-1}}$ where $1<n_1<n_2<\ldots<n_{k-1}$. Let $J(E)=K(E)+p_j$ where j is the first positive integer missing in the series $1, n_1, n_2, \ldots, n_{k-1}, n_{k-1}+1, \ldots, n-1$. Likewise p_2

is the related element to the set $E = p_2, p_{n_1+1}, p_{n_2+1}, \ldots, p_{n_{k-1}+1}$, and its J set contains p_{j+1} and in general p_i is the related element to the set $E = p_i, p_{n_1+i-1}, p_{n_2+i-1}, \ldots, p_{n_{k-1}+i-1}$ and p_{j+i-1} belongs to its J-set. The subscripts are considered as modulo n. Choose another E containing p_1 not already picked in the above group of sets and having the same number k of elements. Define its J-sets in the same manner. There being only a finite number of sets this process can be continued until every set of k elements has an added J-point.

Apply this rule for every k and the resulting space is P5678.

Apply the rule for $k = 2$ only and let every set have J-points including all J-points of its subsets. The result is a space P*234*.

Apply the rule to sets of an even number of elements. The space is P*124*.

The rule can only be applied to spaces having a prime number of elements because it is only in this case that the number of subsets of a given number k of elements is an exact multiple of n. Otherwise if only one element is added as J-point some elements must be used more often and the space is not homogeneous or else more than one element is added by the rule and property five is not present in general.

Spaces of six elements having properties P*234* and P5678 have been shown to be possible while P*124* can be shown to be impossible for six element spaces. These three and P*24*, concerning which we can make no statement, involve the uniqueness property and are connected and accessible. We note elsewhere that for a space of an even number of elements to be connected and homogeneous some subset of n/2 elements must have the entire space as J-set. A difficulty arises in attempting to make this definition and preserve the uniqueness property in the general situation.

Thus we have shown nine of the 32 cases impossible. We have given a method of constructing 19 of the cases for finite spaces having some six or more elements. Three cases have been demonstrated for spaces with a prime number of elements and nothing is known concerning one case.

The section on properties of integers gives some unusual cases arising in the study of the space P*48*. In this connection also we find that if a space of seven elements has K(E) includes E and if E includes p_i, p_{i+1} then K(E) includes p_{i+2} (modulo 7) then the space is connected but can be partitioned into three disjoined parts every two of which have the relation $A\overline{B} + B\overline{A} = O$.

The space P*48* is not only homogeneous according to the definition but is such that for any four elements, a,b,c,d the space can be trans-

formed bicontinuously into itself in such a way that a and b are transformed into c and d. Obviously this extends to any number of elements. Also all of its subsets are homogeneous.

THE SPACE P*34*

Generalizing the homogeneous space S 114 page 101 of my thesis to a finite number *n* of elements we have a P*34* such that if E includes p_i then K(E) includes p_{i+1} modulo n. This is a homogeneous space and we study its continuous transformations into homogeneous spaces. All continuous transformations of the space are connected, singular, perfect, and separable. Its relation to the four remaining properties gives rise to the study of 16 cases.

Theorem 4. *Theorems 1 and 2 are applicable to the transformations of* P*34.*

Proof is similar to the previous proof. A space having properties P*134* is seen to be impossible by this theorem.

Theorem 5. *An accessible continuous transformation of* P*34 has* $J(E) = P$ *for every subset except the null set.*

Only the null set and set P have the property K(K(E)) is included in K(E). Hence J-points must be added for the space to be accessible. If J-sets of single elements have two element J-sets there is no way of adding J-points to sets of two elements so as to have accessibility. The same situation occurs when single elements have any other number of J-points except the number n. Likewise for other subsets.

By theorem 5 the following spaces are impossible; P*4*, P*24*, P*245*, P*14*, P*145*, P*124*, P3678.

The following homogeneous spaces have been defined as continuous transformations of P*34*.

1. Let J(E) = P for every E. P*45*.
2. The identical transformation of P*34*. P*34*.
3. Let J(E) = E + K(E) for every E. P*345*.
4. Let sets of N − 1 elements have J-sets equal P. P*234*.
5. For sets of more than two elements let J(E) = E + K(E). P1678.
6. Let J(E) = E + K(E) and J(O) = P. P2678.
7. Each set of n − 2 consecutive elements (clockwise order) has its first element as J-point. P5678.
8. For sets of two elements only J(E) = E + K(E). P678.

Of the 16 cases to be studied we have found eight possible and have defined them. Theorems have been given to show the impossibility of the other eight. That is, we have shown that the eight properties distinguish only eight transformations of the space P*34*.

OTHER SPACES

The space P*4* studied by Stephens* can be transformed continuously into only one homogeneous space. It has $J(E)=P$ for all E. His space P*47* transforms into this one and into the space with $J(O)=O$ and $J(E)=P$ for $E\neq O$. My thesis shows that both spaces P*4* and P*47* cannot be homogeneous when the number of elements is finite. The homogeneous spaces just mentioned are in general the transformations of any finite space regardless of its properties. The homogeneous space with $K(E)=O$ for all E can be transformed continuously into any homogeneous space having the same number of elements. The number of cases possible cover the range of possibilities for finite homogeneous spaces and has been adequately covered in my thesis.

Homogeneous spaces and integers.

It was noted in my thesis that the number of elements in a cycle of a homogeneous space is a factor of the cardinal number of the space. Prime, odd and even numbers have come to attention frequently in this study. Some of these will be mentioned here.

If finite spaces P*48* are transformed into homogeneous spaces with $J(E)$ includes $K(E)$ and if when E includes $P_i\, p_{i+1}$ we have $J(E)$ includes $p_{,+2}$ (modulo n) then spaces of an odd number of elements are connected and spaces of an even number of elements are not connected. This comes from the fact that in any partition of a space of an odd number of elements at least one of the parts must have two or more consecutive elements.

The definitions of spaces P*234*, P*124* and P5678 as given in a previous section hold for spaces of a prime number of elements only.

Consider a homogeneous space of an even number, n, of elements which is a continuous transformation of a space P*48*. If this space is connected some subset of n/2 elements has $J(E)=P$ where P is the set of all elements of the space.

One is led to ask whether even, odd, and prime numbers can be characterized as being the cardinal numbers of spaces of given properties.

*Stephens, loc. cit. 0. 101.p

"For a finite space of type P*4* consisting of more than one element, the derived set of any set E is given by $K(E)=E+P_1$".

The Teacher's Department

Edited by
JOSEPH SEIDLIN

The following expository article,—"The First Exercise on Differentials," is a fairly complete account of an actual recitation period. To quote Professor Longley: "I feel sure that this is not the worst exercise in the course and it is probably not the best. It has the advantage of being one of the most recent and therefore easiest to write out from memory... The exercise is entirely informal with questions from the students and from the instructor to the class, which are not indicated in the account."

The editor hopes, and believes, that articles such as this will prove to be of value to fellow teachers of mathematics. It is not the intention of the editor or Professor Longley to advance the claim that here is *the only way* or *the best way* of introducing differentials. But here is *a way* employed by a well-known teacher with many years of fine experience in the field.

Some of us may have a method we believe superior to the one suggested by Professor Longley. Be that as it may, the editor and our readers are indebted to Professor Longley for sharing with us his way of doing it.

First Exercise on Differentials

By W. R. LONGLEY
Yale University

If y is a function of x, $y = f(x)$, we have defined the derivative of y with respect to x and have denoted it by various symbols, e. g., dy/dx, y', $f'(x)$. We have insisted that the symbol dy/dx is not an algebraic quotient and have called it an operator, indicating that $y = f(x)$ is to be operated on according to certain specified rules.

Review questions.

What is the definition of derivative?

What is the meaning of Δx, Δy? (Increments).

What is the geometric interpretation of $f'(x)$?

(Slope of the tangent to the curve $y = f(x)$).

What is the general interpretation of the derivative?

(Rate of change of y with respect to x.)

We now, by definition, give a meaning to the symbols dx and dy.

Between dx and Δx there is no distinction and we shall understand that dx denotes a small change in the value of x. In later applications in integration, dx denotes an infinitesimal, that is a variable which approaches zero as a limit. For our immediate purpose, dx will denote a small, but finite, change (either positive or negative) in the value of x. The symbol dx is called the *differential* of x and, assuming always that x is the independent variable, dx has the same significance as the increment Δx.

The meaning of the symbol dy is indicated by the following definition.

The differential of a function of a variable is the product of its derivative with respect to the variable by the increment of the variable.

In symbols, $\quad dy = f'(x)dx$

Thus, if $\quad y = x^2$

$$dy = 2xdx$$

In order to calculate a numerical value of dy, we must assign a numerical value to x and to dx.

We now investigate the meaning of dy.

I. Representation on the graph of $y = f(x)$.

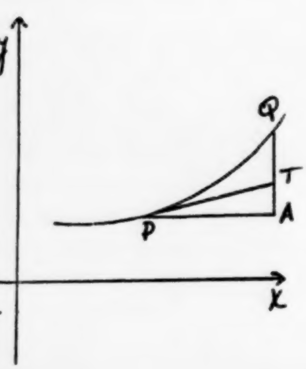


Fig 1.

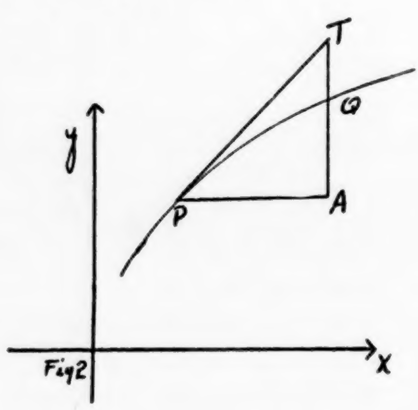
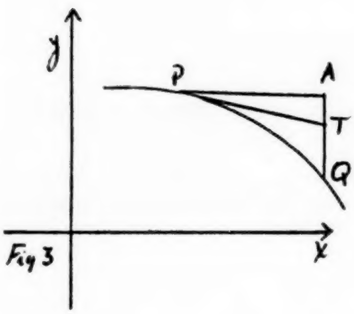
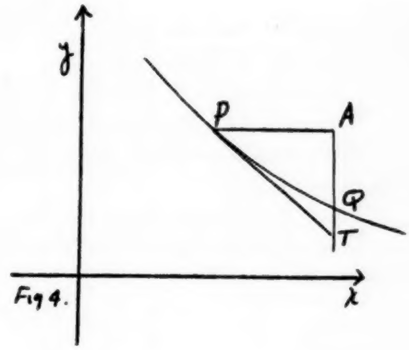
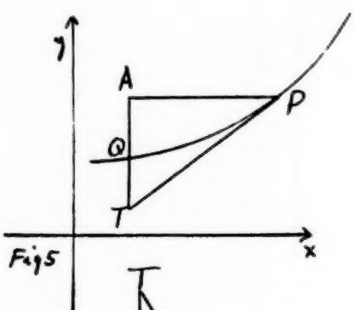
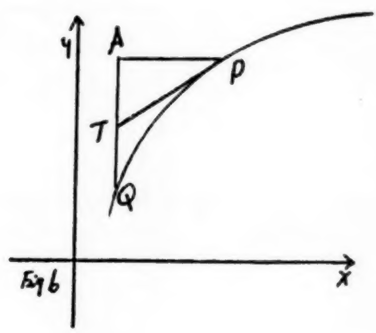


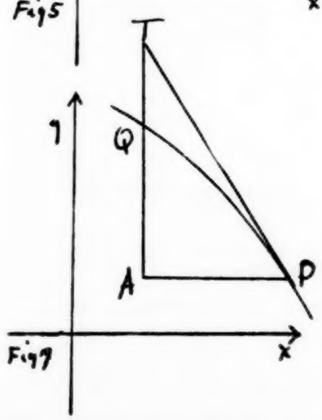
Fig2

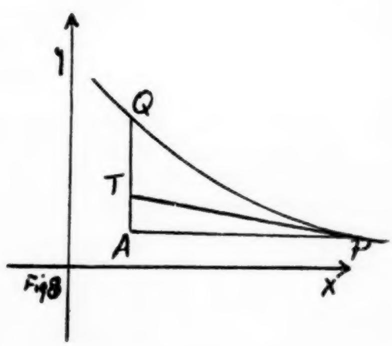












What part of the figure represents Δx? (PA)
What part of the figure represents Δy? (AQ)
What part of the figure represents $f'(x)$?
(Tangent of angle APT or (AT)/(PA))
Referring to the definition, what part of the figure represents dy? (AT)

Thus we have shown that if $P(x,y)$ is a point on the curve $y = f(x)$, then, for a particular value of x and an arbitrarily chosen value of the increment dx, Δy is the corresponding increment of the ordinate drawn

to the curve, and dy is the corresponding increment of the ordinate drawn to the tangent at P.

From the figures we note the following facts.

That dy and Δy may be positive or negative.

That the numerical value of dy may be either larger or smaller than that of Δy.

That $\Delta y - dy = TQ$

That dy, Δy, and (in particular) $\Delta y - dy$ all become smaller when dx becomes smaller.

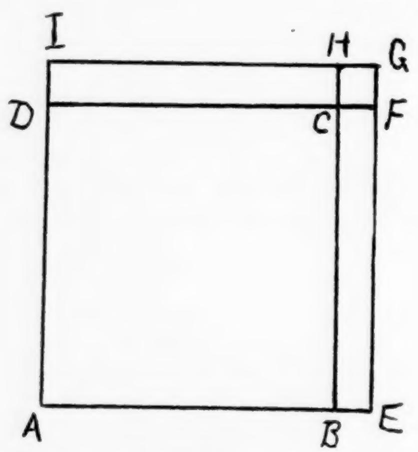


Fig. 9.

II. Consider the special function $K = x^2$, where x represents the length of the side and K the area of a square.

By means of questions bring out the following facts:

When $$x = AB,\ K = ABCD,$$
$$dx = \Delta x = BE = DI,$$
$$dK = BEFC + DCHI,$$
$$\Delta K - dK = CFGH.$$

These facts are shown also algebraically.

$$K = x^2,$$
$$K + \Delta K = (x + \Delta x)^2,$$
$$\Delta K = 2x\Delta x + (\Delta x)^2,$$
$$dK = 2xdx,$$
$$\Delta K - dK = (\Delta x)^2.$$

III. Consider the special function $V = x^3$, from which

$$dV = 3x^2dx.$$

$$\Delta V = 3x^2 \Delta x + 3x(\Delta x)^2 + (\Delta x)^3.$$

Interpret each term of ΔV by means of a cube.

IV. Consider the special function $K = \pi r^2$, from which

$$dK = 2\pi r dr,$$

$$K = 2\pi r \Delta r + \pi(\Delta r)^2.$$

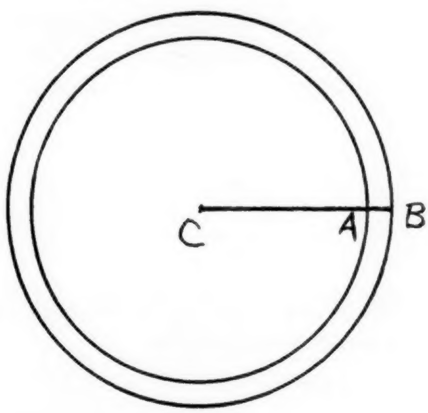


Fig. 10.

When $r = CA$, K = area of the smaller circle.

When $\Delta r = AB, \Delta K$ = area of the ring between the circles, and dK = area of a rectangle with one side equal to dr and the other side equal to $2\pi r$, that is, the circumference of the smaller circle. Make similar statements regarding dr as negative.

If $dr = \Delta r$ is small, the difference $\Delta K - dK = \pi(\Delta r)^2$ is small and the area of the rectangle representing dK is approximately equal to the area of the ring representing ΔK.

Hence the area of a narrow circular ring, of width dr, is given approximately by $C\, dr$, where C denotes the inner (or outer) circumference.

V. Consider the special function $V = 4\pi r^3/_3$, from which $dV = 4\pi r^2dr$. By a discussion similar to that under IV we see that ΔV represents the volume of a spherical shell and that dV is equal to the inner (or outer) surface of the shell multiplied by its thickness.

VI. Consider a particle sliding down a smooth plane inclined at an angle i to the horizontal.

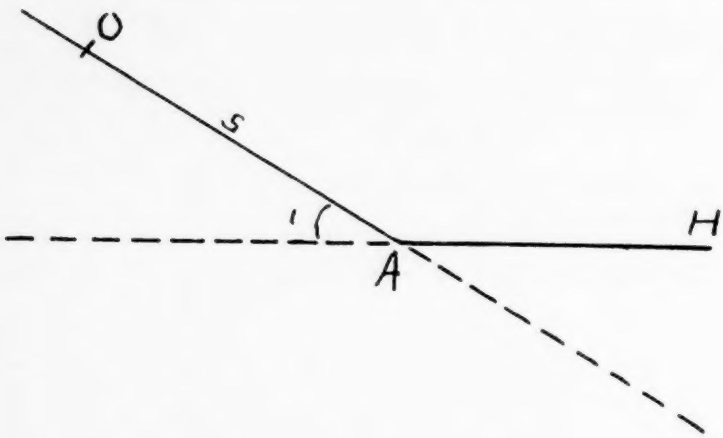


Fig. 11.

Starting from rest at the point O, the distance (s feet) covered by the particle in t seconds is given by

$$s = \tfrac{1}{2}\, gt^2 \sin i.$$

For convenience we will take $g = 32$ and $i = 30°$.

Then $$s = 8t^2$$

The velocity at any instant is given by

$$v = \frac{ds}{dt} = 16t$$

By the definition of the differential,

$$ds = 16t dt = v dt.$$

Suppose the particle reaches the point A at the instant $t = t_1$. It will have an instantaneous velocity $v_1 = 16t_1$.

If the particle continues to slide down the inclined plane, the velocity changes continuously. But if, after it reaches A, the particle slides on a smooth horizontal plane, AH, the velocity does not change and the distance covered is equal to the product of the velocity and the time. We may write

$$ds = v_1 dt.$$

That is, the differential ds represents the distance which would be covered in the interval of time dt if the velocity remains constant ($=v_1$) during this interval.

VII. In Examples IV and V it has been suggested that the value of the differential of the dependent variable may be taken as an approximation to the numerical value of the increment of the dependent variable if the increment of the independent variable is small. This idea may be used to perform certain numerical calculations.

Suppose we wish to calculate the cube root of 990. The result will obviously be nearly equal to the cube root of 1000, which we know is 10. The calculation is performed as follows:

Let (1) $y=x^{1/3}$.

Then (2) $dy=\dfrac{dx}{3x^{2/3}}$.

When $x=1000$ and $dx=-10$, we find from (2),

$$dy=\frac{-10}{300}=-0.033.$$

The calculation may be arranged as follows:

Old value of	$x=1000$	Old value of	$y=10.000$
Change	$=dx=-10$	Change	$=dy=-0.033$
New value	$=x+dx=990$	New value	$=y+dy=9.967$

Hence by the method of differentials $\sqrt[3]{990}=9.967$. A table of cube roots to three decimals gives the same value, 9.967.

The method of differentials gives $\sqrt[3]{970}=9.900$. The correct value to three decimals is 9.899. When $x=1000$ and $dx=-30$, the error in the process above is appreciable in the third decimal place.

An important power not usually given in tables is the $^2/_5$ power. The importance of this exponent comes from the law of physics $p=cv^n$ for a gas confined in a vessel such as steam in the cylinder of an engine. The pressure is denoted by p, the volume by v,

while c and n are constants. For steam at ordinary temperatures the value of n usually taken is 1.4. Then

$$p = cv^{1.4} = cv^{7/5} = cvv^{2/5}.$$

Problem 1. Use differentials to calculate approximately the value of $(30)^{2/5}$.

One important application of differentials is to obtain approximate forumlas which may be used when the exact formulas are cumbersome. Such formulas usually involve trigonometric or other advanced functions, but a simple illustration of the idea is contained in the following problem.

Problem 2. By means of differentials show that, if b is small,

$$\sqrt{a^2+b} = a + \frac{b}{2a}, \text{ approximately.}$$

The remainder of the class period is used by the students in working out the two problems given above.

Mathematical Notes

Edited by

L. J. ADAMS and I. MAIZLISH

Dr. E. J. McShane, formerly of Princeton University, has been appointed Professor of Mathematics at the University of Virginia beginning with the session 1935-36.

A Mathematics Journal Club has been organized at the University of Virginia. Meetings are held bi-weekly, and are devoted to research activities of the staff and students and to reports on articles in the current journals. The speakers for this year have included Professors Whyburn, McShane, Linfield and Luck of the University of Virginia, and Dr. James A. Clarkson, National Research Fellow at Princeton University. The Mathematics Journal Club supplements the activities of the Echols Mathematics Club, which operates under student organization and whose meetings alternate with those of the Journal Club.

Mr. Harry W. Tyler, Consultant in Science for the Library of Congress, comments as follows on a recent item in these pages: *Isis* is the official organ of the History of Science Society, whose headquarters are in this country. It is edited by Dr. George Sarton, a Belgian by birth, who is now connected with Harvard University. The journal is printed in Belgium. The Secretary-Treasurer of the History of Science Society is Mr. F. E. Brasch, of the Library of Congress. The society welcomes the membership of mathematicians who are interested in the history of science and mathematics.

The annual meeting of the Mathematical Association of Great Britain was held at the Institute of Education, London, on January 2, 3, 1936, under the presidency of Mr. A. W. Siddons, of Harrow School, the well known co-author of many mathematical texts. Mr. Siddons, in his presidential address entitled *Progress*, traced the development of the Association from its beginnings in 1871 as the Association for the Improvement of Geometrical Teaching to the present day. He referred in particular to the long struggle which ended in the dethronement of Euclid's *Elements*, for so long the tyrant of school geometry, and gave many details of the change in the manner and purpose of mathematical teaching in the English schools. He concluded with a survey of the present situation and a suggestion that many problems still await their solutions.

Other events of the program of the above meeting included papers on *The Physics of Sport* by Sir Gilbert Walker and on *The Rehabilitation of Differentials* by Professor G. Temple, discussions on *Rider Work in Geometry* and on *University Entrance Scholarships*, and an exhibition of mathematical films. Mr. C. Pendlebury resigned his Secretaryship, an office which he has held since January, 1886. As a mark of the gratitude of the Association for his devoted labours over a period of fifty years as Secretary, he was elected an Honorary Member of the Association.

Dr. Bolling Hall Crenshaw, head of the Department of Mathematics of the Alabama Polytechnic Institute at Auburn, Alabama, died on November 25, 1935. Dr. Crenshaw had served for forty-four years on the Auburn faculty. He was the co-author of five college mathematics texts, was a member of the Mathematical Association of America and was vice-chairman of the executive council of the Institute. Dr. Crenshaw was widely known, and his passing is especially mourned by the hundreds of former students of Alabama Polytechnic Institute, with whom he maintained warm, friendly contact.

The latest directories of the members and officers of the Mathematical Association of America and the American Mathematical Society are contained in the supplements of the December, 1935 and September, 1935 numbers of *The American Mathematical Monthly* and the *Bulletin* of the American Mathematical Society, respectively.

A new slide rule for statisticians has been announced. It is the work of Mr. E. R. Enlow, Director of Statistics and Special Services for the Atlanta, Georgia, Public Schools. It now contains seventeen scales, with the possibility of more being added.

The National Research Council of Japan publishes twelve scientific periodicals, among them being the *Japanese Journal of Mathematics*. The Council has offices at the Imperial Academy House, Ueno Park, Tokyo.

Foyles, one of the world's largest bookstores, has a stock of over three million volumes. Mathematicians will be furnished a catalog of mathematical works, upon application for same. Foyle's is located at 119-125 Charing Cross Road, London, W. C. 2.

Scripta Mathematica announces the early publication of Portfolio I, *Portraits of Eminent Mathematicians*, edited by Professor David Eugene Smith. The first portfolio will contain ten portraits.

A bibliography on the Theory of Linkages, by R. Kanayama, is to be found in the *Tôhoku Mathematical Journal*, Vol. 37, pages 294-319.

The Joint Examinations for admission to Associateship in the Actuarial Society of America or the American Institute of Actuaries are interesting to the mathematics student and professor alike. Although the lines of demarcation of subject matter are not always rigidly drawn, the four parts of the examinations may be roughly classified as follows:

Part 1. College algebra and elementary plane geometry.

Part 2. Trigonometry and analytic geometry.

Part 3. Advanced topics of college algebra, such as systems of higher degree equations, series and choice and chance.

Part 4. Differential and integral calculus, finite differences and statistics.

The problems are generally of a high order of difficulty, with the emphasis naturally being upon those topics with applications in the mathematics of insurance.

The Actuarial Society of America is located at 256 Broadway, New York City, while the offices of the American Institute of Actuaries are at 720 North Michigan Ave., Chicago.

The thirteenth meeting of the Indiana Section of the Mathematical Association of America will be held May 1 and 2, 1936 at Manchester College, North Manchester, Indiana. Following the opening banquet there will be an address by Dr. F. R. Moulton, Director of the Utilities Power and Light Company, Chicago. Dr. Moulton was formerly connected with the Mathematics and Astronomy Departments of the University of Chicago.

The California Institute of Technology has begun a series of coast-to-coast broadcasts on scientific topics. The programs consist of talks by faculty members on popular subjects, which are interspersed with brief dramatizations illustrating the incidents described by the narrator. On February 8, 1936 Professor Eric Temple Bell addressed the radio audience on *Mathematics*. Dramatized incidents on this program included the siege of Syracuse and subsequent death of Archimedes; Newton's solution of the problem of the path of a planet, which was part of his treatment of the problem of motion, and which gave birth to the differential calculus; the duel in which young Galois lost his life at the age of twenty; and Sylvester's prediction that algebraic invariants would some day find their place in the theoretical foundation of chemistry.

L. J. ADAMS.

Problem Department

Edited by
T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

While it is our aim to publish problems of most interest to the readers, it is believed that regular text-book problems are, as a rule, less interesting than others. Therefore, other problems will be given preference when the space for problems is limited.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

SOLUTIONS

No. 87. Proposed by F. A. Rickey, L. S. U.

A diameter AB of a certain circle is extended through A to C so that the length of AC is 3 in. At B, a line segment BD is drawn perpendicular to AB. The length of BD is 9 in. Determine the length of AB so that DC shall be tangent to the circle. Can this problem be solved without the use of algebraic equations of degree higher than the second?

Solution by *G. E. Raynor*, Lehigh University.

Let E be the point of tangency of DC. Then $DE=9$. Also let $EC=y$ and $BO=OA=OE=x$. Lines DO and EA are parallel since both are perpendicular to BE. Hence the segments into which they divide CO and CD are proportional and we have

$$\frac{9}{x}=\frac{y}{3}$$

or

$$xy=27. \tag{1}$$

Also, from triangle OEC,

$$y^2=(x+3)^2-x^2$$

$$2) \qquad = 6x + 9.$$

Adding equations (1) and (2) and transposing we find that

$$y^2 - 36 + xy - 6x = 0,$$

or factoring,

$$(y-6)(x+y+6) = 0.$$

By the nature of the problem the factor $x+y+6$ is obviously positive and hence $y=6$.

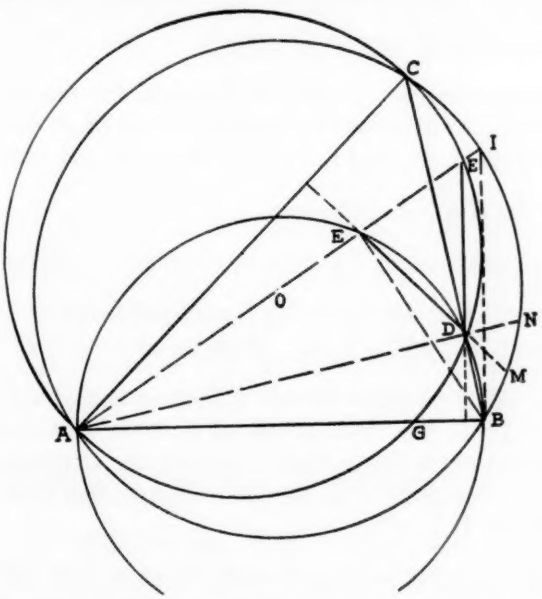
Then from (1) we have

$$AB = 2x = 9 \text{ in.}$$

No. 96. Proposed by Walter B. Clarke.

In a circle with center O, inscribe triangle ABC. Drop a perpendicular from A to D on BC. Drop a perpendicular from B to E on OA. Drop a perpendicular from C to E′ on OA. Show that triangles ABC and DEE′ are equiangular and that DE and DE′ are both perpendicular to sides meeting at A.

Solved by *Karleton W. Crain, Purdue University.*



Extend AO to I, a point on the circumcircle. Draw BI and CI. Let G be the point where the circle whose diameter is AC cuts AB. Draw circles whose diameters are AB and AC respectively.

A,D,E′,C are points on the circle whose diameter is AC. $\angle ACD = \angle EE'D$ (both measured by $\frac{1}{2}$ the arc AGD.) A,B,D,E are points on the circle whose diameter is AB. Now, IB is perpendicular to AB, ($\angle ABI$ inscribed in a semicircle.) and $\angle IBC = \angle BED$ (both measured by arc BD.) But $\angle DEE' = 90° - \angle BED$, and $\angle ABC = 90° - \angle IBC$. Therefore, $\angle DEE' = \angle ABC$. Since two angles of these triangles are respectively equal, the third angle EDE′ must equal angle BAC. And the triangle ABC is similar to triangle DEE′.

To prove E′D perpendicular to AB, observe that $\angle CDE' = \angle CAE'$ (both measured by arc CE′.) $\angle CAE' = \angle IBC$ (both measured by arc CI.) Therefore, $\angle CDE' = \angle IBC$, and DE′ is parallel to BI and so DE′ is perpendicular to AB.

Again, $\angle BCI = \angle BAE$ (both measured by arc BMNI.) $\angle EDC = \angle BAE$ (both measured by arc BDE.) Therefore, $\angle BCI = \angle EDC$, and DE is parallel to IC. But IC is perpendicular to AC, ($\angle ACI$ inscribed in a semi-circle) Therefore, DE is perpendicular to AC.

Also solved by *A. C. Briggs*, Wilmington, Ohio.
R. A. Miller, University of Iowa.
David Blackwell, University of Illinois.

No. 106. Proposed by Walter B. Clarke.

Using the notation: G for centroid, K for Nagel point, I for incenter, and H for orthocenter, prolong IG to M so that GM equals $\frac{1}{2}$ IG. Show that H, M, and K are collinear and that HM equals MK.

Solution by *Henry F. Schroeder*.

Consider points G, H, I and K as referring to triangle ABC.

The lines HI and MO (O being the circumcenter of triangle ABC) are parallel and HI : MO = 2 : 1. (Because HG : GO = 2 : 1.)

The line IK is bisected by O. (Because the nine-point circle of triangle I′ I″ I‴ is the circumcircle of triangle ABC and the center of the nine-point circle lies midway between the circumcenter and the orthocenter.* The point I is the orthocenter and K is the circumcenter of triangle I′ I″ I‴.)

Hence the line HM produced meets IK at K and HM equals MK.
Q. E. D.

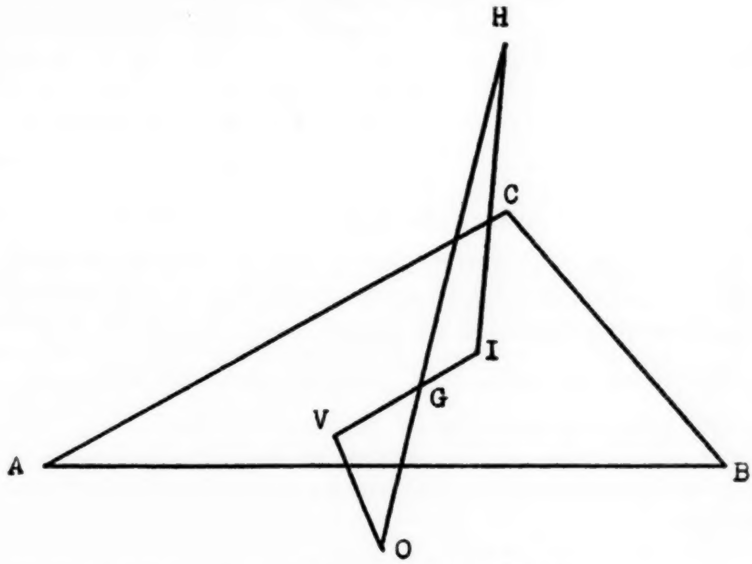
Also solved by *Mrs. A. H. Samuels*, L. S. U.

*These statements are proved on page 94 Altshiller-Court College Geometry.

No. 108. Proposed by Walter B. Clarke.

Using the notation, G for centroid, H for orthocenter, I for incenter, O for circumcenter, and V for verbicenter, (which is the concurrent point of line from each vertex to the point half way around the perimeter,) show that HIG and GOV are equal in area.

Solved by *Carlton W. Crain*, Purdue University.



H, G, and O lie on the Euler line, and $\overline{HG} = 2GO$.

V, G, and I are collinear and $\overline{VG} = 2 \cdot \overline{GI}$. (cf. Johnson, R. A., *Modern Geometry*, pp. 149-225.)

If we let $\overline{GO} = a$ and $\overline{GI} = b$, then the areas of triangles HIG and GOV are equal to ab $\sin\angle HGI$ and ab $\sin\angle VGO$ respectively. Since $\angle HGI = \angle VGO$, the triangles are equal in area.

Also solved by *C. A. Balof* and *Henry Schroeder*.

PROBLEMS FOR SOLUTION

No. 117. Proposed by Walter B. Clarke.

Prove that four circles whose diameters are sides of a cyclic quadrilateral intersect in four concyclic points.

No. 118. Proposed by W. V. Parker, Georgia Tech.

If $f(x)$ is a polynomial of degree $2n+1$, and $g(x)$ is a polynomial of degree $2n$ determined by the conditions

$$f\left[\frac{(2n-k)a+kb}{2n}\right]=g\left[\frac{(2n-k)a+kb}{2n}\right]$$

$$k=0,\ 1,\ldots.2n$$

Then,

$$\int_{a+b/2-\alpha}^{a+b/2+\alpha}\frac{\lambda f(x)+\mu g(x)}{\lambda+\mu}dx=\int_{a+b/2-\alpha}^{a+b/2+\alpha}f(x)dx\qquad(\lambda+\mu\neq 0)$$

No. 119. Proposed by H. T. R. Aude, Colgate University.

If a triangle is placed so that its circumcenter is at the origin and if the coordinates of the vertices are (x_1, y_1), $(x_2\ y_2)$, and (x_3, y_3), the orthocenter is located at the point $(x_1+x_2+x_3,\ y_1+y_2+y_3)$.

No. 120. Proposed by H. T. R. Aude, Colgate University.

The usual interpretation of the radical symbol is that the sign in front of the radical, either written or understood, is the only one taken. It is, therefore, a problem to find the value of k for which the equation

$$\sqrt{x^2-9}+k-x=0$$

has a solution.

Book Reviews

Edited by

P. K. SMITH

Differential Geometry. By W. C. Graustein. The Macmillan Company, New York, 1935; XI+230 pp.

In this book Professor Graustein presents an account of the fundamentals of classical differential geometry. No great mathematical maturity is required of the reader—merely a knowledge of the calculus and some slight familiarity with solid analytical geometry and differential equations. Although the algebra of vectors is employed in the development of the subject the author succeeds in keeping vector analysis in its proper place as a tool.

The first chapter consists of a brief introduction to vector algebra with applications to elementary solid analytical geometry. This is followed, in the next seven chapters, by a connected exposition of the theory; here is taken up the theory of space curves, associated curves and surfaces, the two fundamental forms of a surface, curvature and related topics, the equations of Gauss and Codazzi, geodesics and geodesic curvature, and problems in mapping. The ninth chapter continues with the subject of geodesics, leading the reader from intrinsic properties of geodesic triangles to Levi-Civita's concept of parallelism and closing with a brief introduction to Riemannian geometry. The final chapter is devoted to properties of special surfaces.

This book is a most welcome addition to the elementary literature on geometry. The author's clear style and elegant treatment will win for it very widespread approval. Its content is very well adapted to the needs of students desiring a one semester course in this subject. It seems to the reviewer however that its value to the specialist as well as to the general student would be materially enhanced and would help to create in the reader a broader point of view, had it included more explicit references to the interrelation between differential geometry and other branches of scientific activity. Huygen's study of pendulum clocks and the resulting investigation of evolutes and involutes, Euler's *Mechanics* and its contributions to geodesic theory, the advances arising from the study of problems in cartography and navigation, the various problems in physics which led Monge, Malus, Dupin, Lamé, and so many others to make such noteworthy contri-

butions to geometry, the role of differential geometry in the development of the theory of differential equations are a few examples, mention of which would serve not only to give the student perspective but also to illumine some portions of the general theory.

Typographically the book is unusually excellent. The problems and exercises are chosen with care and are sufficient in number to give the student a good grasp of the subject.

HARRY LEVY,
University of Illinois.

A Mathematician Explains. By Mayme I. Logsdon. University of Chicago Press.

This is a University of Chicago "New Plan" text which surveys the history, content, method, and applications of elementary mathematics through calculus in 175 pages. The author states that an "objective is to look carefully into the *nature of the science* which is commonly labeled 'abstract' and 'deductive' and show that these descriptive terms need not imply that behind them lie mystery and difficulty of comprehension, but rather beauty, elegance, and, above all, *orderliness* and *simplicity*." The examples given are carefully explained with almost no problems left for the reader. The various chapters could be used advantageously as previews of the usual type elementary college courses and would be useful for guidance or orientation of freshmen.

A miniature mathematical system based on four postulates for positive integers is presented in four of the pages in the chapter on arithmetic. Within these pages is given an abstract logical construction of the real number system in a readable style. As a preparation for this there is an interesting treatment of the structure and historical development of our number system together with the properties of the operations and a comparison with other number systems. My students are finding her treatment of change of base most interesting.

Chapter eight, written by Professor Bliss, indicates just how mathematical systems are useful in correlating physical phenomena and predicting new data.

DOROTHY McCOY,
Belhaven College.

Fusion Mathematics. By Aaron Freilich, Chairman, Department of Mathematics, Bushwick High School, Brooklyn, New York; Henry H. Shanholt, Chairman, Department of Mathematics, Abraham

Lincoln High School, Brooklyn, New York; Joseph P. McCormick, Chairman, Department of Mathematics, Theodore Roosevelt High School, Bronx, New York. Cloth, Pages VIII and 600. 13x18 cm. 1934 Silver, Burdett and Company, New York. Price $1.84.

The authors of this text have ignored the traditional division of algebra and trigonometry into two separate subjects, but have skillfully interwoven the subject matter of algebra and trigonometry in each of the chapters of the book. So efficiently has the fusion of these subjects been accomplished that the reviewer agrees with the authors that the student will be unaware that he is studying what heretofore have been two distinct subjects.

In carrying out their plans to make the text self-teaching, touch life situations, and provide for individual differences the authors have not sacrificed any of the essential material of intermediate algebra or any of the essential elements of a first course in plane trigonometry.

Among the desirable features of the book are the historical paragraphs, the grading of the exercises into A, B, C, groups, a new method of finding the characteristic of logarithms, the accumulative reviews, questions for stimulating thought, and the many good reasoning problems.

The seventeen chapters of the text contain a sufficient number of good graphs and figures conveniently placed on the pages. The type is very clear and legible.

It is attractively bound in a light green color.

HENRY SCHROEDER,
Louisiana Polytechnic Institute.

Joint Mathematical Meetings

LOUISIANA-MISSISSIPPI SECTION OF MATHEMATICAL ASSOCIATION OF AMERICA

LOUISIANA-MISSISSIPPI BRANCH OF NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

MISSISSIPPI ACADEMY OF SCIENCE

HATTIESBURG, MISSISSIPPI

FRIDAY AND SATURDAY, MARCH 13 AND 14, 1936

Hosts: Mississippi State Teachers College, Mississippi Woman's College, and Hattiesburg High School.

SCHEDULE:

Friday, March 13, 1936

1:00 P. M. Registration, Administration Building, State Teachers College.

2:00 P. M. Joint Preliminary Meeting, Auditorium.

2:30 P. M. Association Program, Room 36, Science Hall.
Mississippi Academy of Science Program, Auditorium.

5:00 P. M. Reception by Faculty of State Teachers College.

7:30 P. M. Joint Banquet, Dining Hall, Mississippi Woman's College.

Saturday, March 14, 1936.

9:00 A. M. Council Program, Hattiesburg High School.
Mississippi Academy of Science Program, Hattiesburg High School.

11:00 A. M. Separate Business Meetings of the three groups.

HOTEL RATES:

Hotel Hattiesburg: Single Rooms, without bath, \$1.50; Double rooms, without bath, \$2.00; Single rooms, with bath, \$1.75-2.50; Double rooms with bath, \$3.00-3.50.

Leaf Hotel: Single rooms, without bath, \$1.25; Double rooms, without bath, \$1.75; single rooms, with bath, \$1.50-2.25; Double rooms, with bath, \$2.00-3.00.

Forrest Hotel: Single rooms, with bath, \$2.50; Double rooms, with bath, \$4.00-4.50.